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Realizations of Multimode Quantum Group $SU(1,1)_{q,s}$

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By virtue of the two-parameter deformed multimode bosonic oscillator, the Nodvik and Holstein–Primakoff realizations of the two-parameter deformed multimode quantum group $SU(1,1)_{q,s}$ are derived. The deformed mappings between the multimode quantum group $SU(1,1)_{q,s}$ and the two-parameter deformed multimode bosonic oscillators are also presented.

In the past few years, quantum groups and algebras have been shown to arise in many problems of physical and mathematical interest. Much effort is now being devoted to the construction of their representations, and recently many realizations have been usefully devised using the q-deformation and q,s-deformation of single-mode bosonic operators and the q,s-deformation of multimode bosonic oscillators (Faddeev, 1981; Drinfeld, 1986; Jimbo, 1986; Kulish *et al.*, 1981; Biedenharn, 1989; Macfarlane, 1989; Sun *et al.*, 1989; Yan, 1990; Ng, 1990; Katriel *et al.*, 1991; Nodvik, 1969; Demidov *et al.*, 1990; Sudbery, 1990; Schirrmacher *et al.*, 1991; Burdik *et al.*, 1991; Chakrabarti *et al.*, 1991; Jing, 1993; Zhou *et al.*, 1995; Curtright *et al.*, 1990; Song, 1990; Quesne, 1991; Mallick *et al.*, 1991.

Based on our recent work (Yu *et al.*, 1998), the present paper derives the Nodvik and Holstein–Primakoff realizations of the multimode quantum group $SU(1,1)_{q,s}$ and gives the deformed mappings between the multimode quantum group $SU(1,1)_{q,s}$ and the q,s-deformed multimode bosonic oscillators.

We introduce four independent groups of the q,s-deformed bosonic oscillators $\{a_i^\dagger, a_i, n_i^a\}$, $\{b_i^\dagger, b_i, n_i^b\}$, $\{c_i^\dagger, c_i, n_i^c\}$, and $\{d_i^\dagger, d_i, n_i^d\}$ (for $i = 1, 2, \ldots, k$). They satisfy the commutation relations (Jing, 1993)

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$$a_i^\dagger a_i = [n_i^a]_{qs}, \qquad a_i a_i^\dagger = [n_i^a + 1\,]_{qs},$$

$$[n_i^a, a_i^\dagger] = a_i^\dagger, \qquad [n_i^a, a_i] = -a_i \tag{1a}$$

$$a_i a_i^\dagger - s^{-1} q a_i^\dagger a_i = (sq)^{-n_i^a}, \qquad a_i a_i^\dagger - (sq)^{-1} a_i^\dagger a_i = (s^{-1}q)^{n_i^a} \tag{1b}$$

$$b_i^\dagger b_i = [n_i^b]_{qs^{-1}}, \qquad b_i b_i^\dagger = [n_i^b + 1]_{qs^{-1}},$$

$$[n_i^b, b_i^\dagger] = b_i^\dagger, \qquad [n_i^b, b_i] = -b_i \tag{2a}$$

$$b_i b_i^\dagger - sq b_i^\dagger b_i = (sq^{-1})^{n_i^b} \tag{2b}$$

$$c_i^\dagger c_i = [n_i^c]_{qs}, \qquad c_i c_i^\dagger = [n_i^c + 1]_{qs},$$

$$[n_i^c, c_i^\dagger] = c_i^\dagger, \qquad [n_i^c, c_i] = -\, c_i \tag{3a}$$

$$c_i c_i^+ - s^{-1} q c_i^+ c_i = (sq)^{-n_i^c}, \qquad c_i c_i^+ - (sq)^{-1} c_i^+ c_i = (s^{-1}q)^{n_i^c} \tag{3b}$$

$$d_i^+ d_i = [n_i^d]_{qs^{-1}}, \qquad d_i d_i^+ = [n_i^d + 1]_{qs^{-1}},$$

$$[n_i^d, d_i^+] = d_i^+, \qquad [n_i^d, d_i] = -d_i \tag{4a}$$

$$d_i d_i^+ - sq d_i^+ d_i = (sq^{-1})^{n_i^d} \tag{4b}$$

where we have used the notations $[x]_{q,s} = s^{1-x}(q^x - q^{-x})/(q - q^{-1})$; the x can be operators or general numbers.

Similar to our former work (Yu *et al.*, 1998), we define four independent q,s-deformed k-mode bosonic operators as follows:

$$A_k = a_1 a_2 \ldots a_k \left\{ \frac{[n_1^a]_{qs}[n_2^a]_{qs} \ldots [n_k^a]_{qs}}{\min([n_1^a]_{qs}, [n_2^a]_{qs}, \ldots, [n_k^a]_{qs})} \right\}^{-1/2} \tag{5}$$

$$B_k = b_1 b_2 \ldots b_k \left\{ \frac{[n_1^b]_{qs^{-1}}[n_2^b]_{qs^{-1}} \ldots [n_k^b]_{qs^{-1}}}{\min([n_1^b]_{qs^{-1}}, [n_2^b]_{qs^{-1}}, \ldots, [n_k^b]_{qs^{-1}})} \right\}^{-1/2} \tag{6}$$

$$C_k = c_1 c_2 \ldots c_k \left\{ \frac{[n_1^c]_{qs}[n_2^c]_{qs} \ldots [n_k^c]_{qs}}{\min([n_1^c]_{qs}, [n_2^c]_{qs}, \ldots, [n_k^c]_{qs})} \right\}^{-1/2} \tag{7}$$

$$D_k = d_1 d_2 \ldots d_k \left\{ \frac{[n_1^d]_{qs^{-1}}[n_2^d]_{qs^{-1}} \ldots [n_k^d]_{qs^{-1}}}{\min([n_1^d]_{qs^{-1}}, [n_2^d]_{qs^{-1}}, \ldots, [n_k^d]_{qs^{-1}})} \right\}^{-1/2} \tag{8}$$

It is easy to check the following:

$$A_k A_k^+ - s^{-1} q A_k^+ A_k = (sq)^{-N_k^a}, \qquad A_k A_k^+ - (sq^{-1}) A_k^+ A_k = (s^{-1}q)^{N_k^a} \tag{9a}$$

$$[N_k^a, A_k^+] = A_k^+, \qquad [N_k^a, A_k] = -A_k \tag{9b}$$

$$B_k B_k^+ - sq B_k^+ B_k = (sq^{-1})^{N_k^b} \tag{10a}$$

$$[N_k^b, B_k^+] = B_k^+, \qquad [N_k^b, B_k] = -B_k \tag{10b}$$

$$C_k C_k^+ - s^{-1} q C_k^+ C_k = (sq)^{-N_k^c}, \qquad C_k C_k^+ - (sq^{-1}) C_k^+ C_k = (s^{-1}q)^{N_k^c} \tag{11a}$$

$$[N_k^c, C_k^+] = C_k^+, \qquad [N_k^c, C_k] = -C_k \tag{11b}$$

$$D_k D_k^+ - sq D_k^+ D_k = (sq^{-1})^{N_k^d} \tag{12a}$$

$$[N_k^d, D_k^+] = D_k^+, \qquad [N_k^d, D_k] = -D_k \tag{12b}$$

where N_k^a, N_k^b, N_k^c, and N_k^d are given by

$$N_k^a = \min(n_1^a, n_2^a, \ldots, n_k^a) \tag{13}$$

$$N_k^b = \min(n_1^b, n_2^b, \ldots, n_k^b) \tag{14}$$

$$N_k^c = \min(n_1^c, n_2^c, \ldots, n_k^c) \tag{15}$$

$$N_k^d = \min(n_1^d, n_2^d, \ldots, n_k^d) \tag{16}$$

It is easy to find that $\{A_k^+, A_k, N_k^a\}$, $\{B_k^+, B_k, N_k^b\}$, $\{C_k^+, C_k, N_k^c\}$, and $\{D_k^+, D_k, N_k^d\}$ indicate q,s-deformed k-mode bosonic oscillators, respectively.

Similar to the single-mode quantum group $SU(1,1)_{q,s}$ (Jing, 1993; Jing *et al.*, 1993), the k-mode quantum group $SU(1,1)_{q,s}$ also has three representations: (a) positive discrete series; (b) negative discrete series; (c) continuous series, which we do not consider here. The generators of the k-mode quantum group $SU(1,1)_{q,s}$ can be obtained from a Jordan–Schwinger realization in terms of the q,s-deformed k-mode bosonic oscillator creation and annihilation operators

$$(L_k^{(a)})_+ = s^{-1} A_k^+ C_k^+, \qquad (L_k^{(a)})_- = s^{-1} C_k A_k,$$

$$(L_k^{(a)})_0 = \frac{1}{2}(N_{k,a}^{(a)} + N_{k,c}^{(a)} + 1) \tag{17a}$$

$$(L_k^{(b)})_+ = s B_k D_k, \qquad (L_k^{(b)})_- = s D_k^+ B_k^+,$$

$$(L_k^{(b)})_0 = \frac{-1}{2}(N_{k,b}^{(b)} + N_{k,d}^{(b)} + 1) \tag{17b}$$

where the notations are defined by

$$N_{k,a}^{(a)} = N_k^a, \qquad N_{k,b}^{(b)} = N_k^b, \qquad N_{k,c}^{(a)} = N_k^c, \qquad N_{k,d}^{(b)} = N_k^d \tag{18}$$

It is easy to check that equations (17a) and (17b) satisfy the following commutation relations:

$$[(L_k^{(a)})_0, (L_k^{(a)})_\pm] = \pm(L_k^{(a)})_\pm \tag{19a}$$

$$s^{-1}(L_k^{(a)})_+(L_k^{(a)})_- - s(L_k^{(a)})_-(L_k^{(a)})_+ = s^{-2(L_k^{(a)})_0}[2(L_k^{(a)})_0] \tag{19b}$$

$$[(L_k^{(\mathrm{b})})_0, (L_k^{(\mathrm{b})})_\pm] = \pm(L_k^{(\mathrm{b})})_\pm \tag{20a}$$

$$s^{-1}(L_k^{(\mathrm{b})})_+(L_k^{(\mathrm{b})})_- - s(L_k^{(\mathrm{b})})_-(L_k^{(\mathrm{b})})_+ = s^{-2(L_k^{(\mathrm{b})})_0}[2(L_k^{(\mathrm{b})})_0] \tag{20b}$$

The k-mode quantum group $\mathrm{SU}(1,1)_{q,s}$ is a Hopf algebra; its coproduct, antipode, and counit are, respectively, as follows.

Coproduct:

$$\Delta((L_k^{(\mathrm{a})})_0) = (L_k^{(\mathrm{a})})_0 \otimes 1 + 1 \otimes (L_k^{(\mathrm{a})})_0 \tag{21a}$$

$$\Delta((L_k^{(\mathrm{a})})_\pm) = (L_k^{(\mathrm{a})})_\pm \otimes (sq)^{-(L_k^{(\mathrm{a})})_0} + (s^{-1}q)^{(L_k^{(\mathrm{a})})_0} \otimes (L_k^{(\mathrm{a})})_\pm \tag{21b}$$

$$\Delta(1) = 1 \otimes 1 \tag{21c}$$

$$\Delta((L_k^{(\mathrm{b})})_0) = (L_k^{(\mathrm{b})})_0 \otimes 1 + 1 \otimes (L_k^{(\mathrm{b})})_0 \tag{22a}$$

$$\Delta((L_k^{(\mathrm{b})})_\pm) = (L_k^{(\mathrm{b})})_\pm \otimes (sq)^{-(L_k^{(\mathrm{b})})_0} + (s^{-1}q)^{(L_k^{(\mathrm{b})})_0} \otimes (L_k^{(\mathrm{b})})_\pm \tag{22b}$$

$$\Delta(1) = 1 \otimes 1 \tag{22c}$$

Antipode:

$$S((L_k^{(\mathrm{a})})_0) = -(L_k^{(\mathrm{a})})_0 \tag{23a}$$

$$S((L_k^{(\mathrm{a})})_+) = -(sq^{-1})(L_k^{(\mathrm{a})})_+ s^{2(L_k^{(\mathrm{a})})_0} \tag{23b}$$

$$S((L_k^{(\mathrm{a})})_-) = -(sq^{-1})(L_k^{(\mathrm{a})})_- s^{2(L_k^{(\mathrm{a})})_0} \tag{23c}$$

$$S((L_k^{(\mathrm{b})})_0) = -(L_k^{(\mathrm{b})})_0 \tag{24a}$$

$$S((L_k^{(\mathrm{b})})_+) = -(sq^{-1})(L_k^{(\mathrm{b})})_+ s^{2(L_k^{(\mathrm{b})})_0} \tag{24b}$$

$$S((L_k^{(\mathrm{b})})_-) = -(sq^{-1})(L_k^{(\mathrm{b})})_- s^{2(L_k^{(\mathrm{b})})_0} \tag{24c}$$

Counit:

$$\epsilon((L_k^{(a)})_0) = \epsilon((L_k^{(a)})_\pm) = 0 \tag{25a}$$

$$\epsilon(1) = 1 \tag{25b}$$

$$\epsilon((L_k^{(b)})_0) = \epsilon((L_k^{(b)})_\pm) = 0 \tag{26a}$$

$$\epsilon(1) = 1 \tag{26b}$$

The two discrete unitary irreducible representations of the k-mode quantum group $\mathrm{SU}(1,1)_{q,s}$ are

$$|l, r; l, r; \ldots\rangle^a = |r - l - 1; r - l - 1; \ldots\rangle^a \otimes |r + l; r + l; \ldots\rangle^a \qquad (r \geq -l > 0) \tag{27a}$$

$$|l, r; l, r; \ldots\rangle^b = |-r - l - 1; -r - l - 1; \ldots\rangle^b \bigotimes |-r + l; -r + l; \ldots\rangle^b \qquad (r \le l < 0) \tag{27b}$$

These irreducible representations are infinite dimensional and depend on the quantum numbers $l = -1/2, -1, \ldots$. The action of the k-mode quantum group $SU(1,1)_{q,s}$ generators on the elements of the irreducible representations (27a) and (27b) is given by

$$(L_k^{(a)})_+|l, r; l, r; \ldots\rangle^a = s^{-1}\sqrt{[r-l]_{qs}[r+l+1]_{qs}}|l, r+1; l, r+1; \ldots\rangle^a \tag{28a}$$

$$(L_k^{(a)})_-|l, r; l, r; \ldots\rangle^a = s^{-1}\sqrt{[r+l]_{qs}[r-l-1]_{qs}}|l, r-1; l, r-1; \ldots\rangle^a \tag{28b}$$

$$(L_k^{(a)})_0|l, r; l, r; \ldots\rangle^a = r|l, r; l, r; \ldots\rangle^a \tag{28c}$$

and

$$(L_k^{(b)})_+|l, r; l, r; \ldots\rangle^b = s\sqrt{[-r-l-1]_{qs^{-1}}[-r+l]_{qs^{-1}}}|l, r+1; l, r+1; \ldots\rangle^b \tag{29a}$$

$$(L_k^{(b)})_-|l, r; l, r; \ldots\rangle^b = s\sqrt{[-r-l]_{qs^{-1}}[-r+l+1]_{qs^{-1}}}|l, r-1; l, r-1; \ldots\rangle^b \tag{29b}$$

$$(L_k^{(b)})_0|l, r; l, r; \ldots\rangle^b = r|l, r; l, r; \ldots\rangle^b \tag{29c}$$

The Casimir operators of the k-mode quantum group $SU(1,1)_{q,s}$ are

$$C^{(a)} = s^{2(L_k^{(a)})_0}\{-s^2(L_k^{(a)})_-(L_k^{(a)})_+ + [(L_k^{(a)})_0]_{qs}[(L_k^{(a)})_0 + 1]_{qs}\} \tag{30a}$$

$$C^{(b)} = s^{2(L_k^{(b)})_0}\{-(L_k^{(b)})_-(L_k^{(b)})_+ + s^2[-(L_k^{(b)})_0]_{qs^{-1}}[-(L_k^{(b)})_0 - 1]_{qs^{-1}}\} \tag{30b}$$

According to the above properties of the k-mode quantum group $SU(1,1)_{q,s}$, it is easy to obtain its Nodvik realizations as follows:

$$(L_k^{(a)})_+ = s^{-1}e^{-ip_k^a}\sqrt{[l + u_k^a]_{qs}[u_k^a - l + 1]_{qs}} \tag{31a}$$

$$(L_k^{(a)})_- = s^{-1}\sqrt{[l + u_k^a]_{qs}[u_k^a - l + 1]_{qs}}e^{ip_k^a} \tag{31b}$$

$$(L_k^{(a)})_0 = u_k^a \tag{31c}$$

and

$$(L_k^{(b)})_+ = s\sqrt{[l + u_k^b]_{qs^{-1}}[u_k^b - l + 1]_{qs^{-1}}}e^{ip_k^b} \tag{32a}$$

$$(L_k^{(b)})_- = se^{-ip_k^b}\sqrt{[l + u_k^b]_{qs^{-1}}[u_k^b - l + 1]_{qs^{-1}}} \tag{32b}$$

$$(L_k^{(b)})_0 = -u_k^b \tag{32c}$$

where $\{u_k^a, p_k^a\}$ and $\{u_k^b, p_k^b\}$ are the canonical commutators, namely,

$$[u_k^a, p_k^a] = i, \qquad [u_k^b, p_k^b] = i \tag{33}$$

Equations (31) and (32) hold equations (19) and (20), respectively.

For the q,s-deformed k-mode bosonic oscillators we have the the following new realizations: (1) For the positive discrete series (a):

$$A_k = \sqrt{[u_k^a - l + 1]_{qs}}e^{ip_k^a}, \qquad A_k^+ = e^{-ip_k^a}\sqrt{[u_k^a - l + 1]_{qs}} \quad (34a)$$

$$A_k^+ A_k = [u_k^a - l]_{qs} \quad (34b)$$

or

$$C_k = \sqrt{[u_k^a - l + 1]_{qs}}e^{ip_k^a}, \qquad C_k^+ = e^{-ip_k^a}\sqrt{[u_k^a - l + 1]_{qs}} \quad (35a)$$

$$C_k^+ C_k = [u_k^a - l]_{qs} \quad (35b)$$

(2) For the negative discrete series (b):

$$B_k = \sqrt{[u_k^b - l + 1]_{qs^{-1}}}e^{ip_k^b}, \qquad B_k^+ = e^{-ip_k^b}\sqrt{[u_k^b - l + 1]_{qs^{-1}}} \quad (36a)$$

$$B_k^+ B_k = [u_k^b - l]_{qs^{-1}} \quad (36b)$$

or

$$D_k = \sqrt{[u_k^b - l + 1]_{qs^{-1}}}e^{\mathrm{i}p_k^b}, \qquad D_k^+ = e^{-\mathrm{i}p_k^b}\sqrt{[u_k^b - l + 1]_{qs^{-1}}} \quad (37a)$$

$$D_k^+ D_k = [u_k^b - l]_{qs^{-1}} \quad (37b)$$

Therefore we have the deformed mappings between the k-mode quantum group $SU(1,1)_{q,s}$ and the q,s-deformed k-mode bosonic oscillators operators with tildes indicate nondeformed cases):

(1) For the positive discrete series (a):

$$(L_k^{(a)})_+ = s^{-1}(\widetilde{L_k^{(a)}})_+ f((L_k^{(a)})_0), \qquad (L_k^{(a)})_- = s^{-1} f((L_k^{(a)})_0)(\widetilde{L_k^{(a)}})_- \quad (38a)$$

$$(L_k^{(a)})_0 = (\widetilde{L_k^{(a)}})_0 \quad (38b)$$

and

$$A_k = \tilde{A}_k\sqrt{\frac{[N_k^a]_{qs}}{N_k^a}}, \qquad A_k^+ = \sqrt{\frac{[N_k^a]_{qs}}{N_k^a}}\tilde{A}_k^+ \quad (39a)$$

or

$$C_k = \tilde{C}_k\sqrt{\frac{[N_k^c]_{qs}}{N_k^c}}, \qquad C_k^+ = \sqrt{\frac{[N_k^c]_{qs}}{N_k^c}}\tilde{C}_k^+ \quad (39b)$$

where

$$(\widetilde{L_k^{(a)}})_+ = e^{-\mathrm{i}p_k^a}\sqrt{(l + (L_k^{(a)}))_0((L_k^{(a)})_0 - l + 1)} \quad (40)$$

$$(\widetilde{L_k^{(a)}})_- = \sqrt{(l + (L_k^{(a)})_0)((L_k^{(a)})_0 - l + 1)}e^{\mathrm{i}p_k^a}, \qquad (\widetilde{L_k^{(a)}})_0 = u_k^a \quad (41)$$

$$f((L_k^{(a)})_0) = \sqrt{\frac{[l + (L_k^{(a)})_0]_{qs}[(L_k^{(a)})_0 - l + 1]_{qs}}{(l + (L_k^{(a)})_0)((L_k^{(a)})_0 - l - 1)}} \tag{42}$$

(2) For the negative discrete series (b):

$$(L_k^{(b)})_+ = s(\widetilde{L_k^{(b)}})_+ f((L_k^{(b)})_0), \qquad (L_k^{(b)})_- = sf((L_k^{(b)})_0)(\widetilde{L_k^{(b)}})_- \tag{43a}$$

$$(L_k^{(b)})_0 = (\widetilde{L_k^{(b)}})_0 \tag{43b}$$

and

$$B_k = \tilde{B}_k \sqrt{\frac{[N_k^b]_{qs^{-1}}}{N_k^b}}, \qquad B_k^\dagger = \sqrt{\frac{[N_k^b]_{qs^{-1}}}{N_k^b}}\, \tilde{B}_k^\dagger \tag{44a}$$

or

$$D_k = \tilde{D}_k \sqrt{\frac{[N_k^d]_{qs^{-1}}}{N_k^d}}, \qquad D_k^\dagger = \sqrt{\frac{[N_k^d]_{qs^{-1}}}{N_k^d}}\, \tilde{D}_k^\dagger \tag{44b}$$

where

$$(\widetilde{L_k^{(b)}})_+ = e^{-ip_k^b} \sqrt{(-(L_k^{(b)})_0 - l - 1)(-(L_k^{(b)})_0 + l)} \tag{45}$$

$$(\widetilde{L_k^{(b)}})_- = \sqrt{(-(L_k^{(b)})_0 - l - 1)(-(L_k^{(b)})_0 + l)}\, e^{ip_k^b}, \qquad (\widetilde{L_k^{(b)}})_0 = -u_k^{(b)} \tag{46}$$

$$f((L_k^{(b)})_0) = \sqrt{\frac{[-(L_k^{(b)})_0 - l - 1]_{qs^{-1}}[-(L_k^{(b)})_0 + l]_{qs^{-1}}}{(-(L_k^{(b)})_0 - l - 1)(-(L_k^{(b)})_0 + l)}} \tag{47}$$

In order to obtain the Holstein–Promakoff realizations of the k-mode quantum group SU(1,1)$_{q,s}$, its generators can be represented by the q,s-deformed k-mode bosonic oscillators: (1) For the positive discrete series (a):

$$(L_k^{(a)})_+ = s^{-1} A_k^+ \sqrt{[2l + N_k^a]_{qs}}, \qquad (L_k^{(a)})_- = s^{-1} \sqrt{[2l + N_k^a]_{qs}} A_k \tag{48a}$$

$$(L_k^{(a)})_0 = l + N_k^a \tag{48b}$$

or

$$(L_k^{(a)})_+ = s^{-1} C_k^\dagger \sqrt{[2l + N_k^c]_{qs}}, \qquad (L_k^{(a)})_- = s^{-1} \sqrt{[2l + N_k^c]_{qs}} C_k \tag{49a}$$

$$(L_k^{(a)})_0 = l + N_k^c \tag{49b}$$

(2) For the negative discrete series (b):

$$(L_k^{(b)})_+ = s \sqrt{[2l + N_k^b]_{qs^{-1}}} B_k, \qquad (L_k^{(b)})_- = sB_k^+ \sqrt{[2l + N_k^b]_{qs^{-1}}} \tag{50a}$$

$$(L_k^{(b)})_0 = -\,(l + N_k^b) \tag{50b}$$

or

$$(L_k^{(b)})_+ = s\sqrt{[2l + N_k^d]_{qs^{-1}}}D_k, \qquad (L_k^{(b)})_- = sD_k^+\sqrt{[2l + N_k^d]_{qs^{-1}}} \quad (51a)$$

$$(L_k^{(b)})_0 = -(l + N_k^d) \quad (51b)$$

Equations (48)–(51) are the Holstein–Primakoff realizations of the q,s-deformed k-mode quantum group $SU(1,1)_{q,s}$.

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